

LETTERS TO THE EDITOR

Actuarial and Kaplan-Meier survival analysis: There is a difference

To the Editor:

While attending the Seventy-ninth Annual Meeting of The American Association for Thoracic Surgery in New Orleans, I was dismayed to see the almost universal misuse of the term “actuarial survival.”

There are two general types of analysis for survival information: actuarial and Kaplan-Meier. An actuarial analysis should be performed when the actual date of a survival event is unknown. The known information is that the event occurred between time t_n and time t_{n+1} . Actuarial analysis is carried out at specific time intervals (6 months, 1 year), and the resulting graph will step only at those intervals. As the actual failure time is only approximated by the end point, the convention is to attribute a survival time of the fully completed intervals plus half the time of the interval during which the event occurred. Examples of events needing actuarial analysis include population-based death rates and disease-free survival.

Kaplan-Meier analysis is used when the actual date of the end point is known. End points not reached are treated as censored at the date of last follow-up for the analysis. Kaplan-Meier analysis is undertaken at each survival event, death, or censoring, and the graphs will step at each failure time and may or may not be drawn to show the location of censored observations. Examples of an appropriate event for Kaplan-Meier analysis would be postoperative survival when the date of deaths is known.

Most of the misuse of the term “actuarial survival” came during presentation of data with well-known end points and graphs that clearly reflected a Kaplan-Meier analysis. To mislabel the precise survival estimates with the term “actuarial survival” suggests a limited understanding of survival analysis.

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Reply to the Editor:

We are indebted to Dr Wormuth for drawing attention to inaccurate terminology related to so-called survival analysis that has crept into our Association’s annual meeting (and into submitted manuscripts). Communication is the essence of such meetings and of the *Journal*; yet communication controversies continue even within cardiothoracic surgery in the area of medical terminology. These pale, however, in comparison with those in the arena of survival analysis. When presenters and authors deem it necessary to select an adjective

to qualify their estimates of time-related events, they find no universally accepted term! Even the general name for the analyses performed is not agreed on, in part because the methodology is widely applicable: survival analysis, survival modeling, survival data analysis, actuarial analysis, life table, life-history analysis, life data analysis, life-testing, failure-time data, event-history analysis, and censored data analysis (I may have missed some). We see survival analysis in the narrow context of death and other time-related morbid events in cardiothoracic surgery; however, historically, it has roots in other disciplines that continue to contribute to its development, including demography, annuities and insurance, and life-history of machinery in industry. These are further generalized to competing risks, multiple decrement, Markov process, and other names, not to mention the host of names associated with biomathematical models! The terms have come from different, often independent, historical roots, but they all relate to the general mathematical and statistical theory of counting processes (martingales).

History. The word *actuarial* comes from the Latin *actuar-ius*, secretary of accounts. The most notable actuary was the Praetorian Prefect Domitius Ulpianus, who produced a table of annuity values early in the 3rd century AD.¹ This table continued to be used in Europe through the 18th century and even into the early 19th. With the emergence of both solid population data and the science of probability, modern so-called life tables were produced by Edmund Halley² (of comet fame) in 1693. He was motivated, as was the actuary Ulpianus, by economics as related to human survival (annuities, life insurance). Workers in this combined area of demography and economics came to be called *actuaries* in the late 18th century. Importantly for this discussion, the methodology of the actuary varied widely. In the 19th century the actuary of the Alliance of London, Benjamin Gompertz,³ developed physiologically based mathematical models of the dynamic human processes of birth and death to characterize survival. This model-based, completely parametric (equations with constants estimated from data) methodology was substantially different from the simple empiric counting methodology of Halley.

In the 300 years since Halley, a multitude of methods has been developed, and often reinvented, in actuarial science, demography, statistics, industry, and medical science. They all have the common goal of estimating the distribution of the intervals between a designated time zero and the occurrence of an event. In modern times, they also imply a suite of methodology applicable to incomplete data. That is, they permit estimates of at least portions of the distribution to be made when, for many subjects, the time of the event’s occurrence is only known to be beyond the last time of observation (so-called right censoring, one of several types of incomplete data).

Estimators versus adjectives. With this background I come to the crux of Dr Wormuth’s concerns. When authors